Multipath Mitigation Using LMS Filtering Techniques

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***Abstract*— Multipath is a phenomenon that occurs when signals reach a receiver by multiple paths, causing constructive or destructive interference that can distort the received signal from what was intended to be transmitted. This is specifically a big issue in wireless communications, most notably in applications such as Global Positioning Systems (GPS) and telecommunication systems among many others. In this paper, we take a look at adaptive filtering techniques that can be used to mitigate the effects of multipath interference. Computer simulations are performed to emulate a pair of radio wave signals, one direct and one multipath, which are then combined to form a received signal. This is then passed through the LMS and NLMS filter to produce an output that is a close replica of the direct signal. Plots of the squared error of the signal and the convergence of the weights of the filters are looked at to evaluate filter performance. The results proved to be very promising, with the error going to zero very quickly, as well as fast convergence of the weights.**

***Key Words* — Multipath, LMS, NLMS**

I. Introduction

In wireless communications, one of the biggest problems that is faced is interference caused by multipath. It is a propagation phenomenon that results in the distortion of the transmitted signal from the original signal. When a base station of a communication system transmits a signal, it travels through multiple paths to reach its destination. Each waveform experiences attenuation and undergoes phase delays along the way, due to reflection and/or refraction. For example, when travelling through the ionosphere, signals are known to be both reflected and refracted; signals are also reflected when they strike terrestrial objects such as mountains. When these signals reach the receiver, they superimpose on each other, resulting in the received signal to be different from what was intended to be transmitted. This distortion leads to errors in communication, which can have very problematic outcomes. For example, in Global Positioning Systems (GPS), it can lead to inaccurate readings of position. Another system that suffers from multipath is the passive radar system. Passive radar is used in all kinds of media – radio, television, and even real time communications such as GSM. Thus, mitigating multipath is a research area of great significance.

There have been a couple of ways that multipath mitigation has been done so far: a) site location, b) hardware, c) software, and d) a hybrid combination of both hardware and software. Site location refers to choosing a location for the antenna where multipath occurs the least amount. Hardware configurations include methods such as using specialized antenna designs, and using special materials that can absorb microwaves, among other things. Software techniques involve filtering algorithms such as the Least Mean Squares (LMS) and Recursive Least Squares (RLS) techniques. The best results are achieved when a combination of all the techniques mentioned are used.

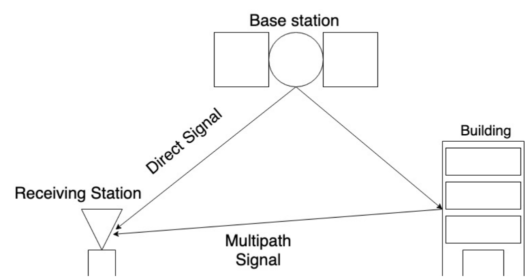

Fig. 1. Simplistic view of multipath propagation. The direct signal and the multipath signal combine at the receiving station, making the received signal distorted.

II. Technical Approach: Signal modelling

In this paper, a study is done on adaptive filtering techniques that can be used to mitigate the effects of multipath to recover the intended signal. The first filter that is looked at is the generic version of the LMS filter. The goal is to simulate radio wave signals that are modelled after real world transmitted signals. According to the Federal Communications Commission (FCC), the range of frequency that has been allocated to be used for radio transmissions is 9 kHz to 275 GHz. Thus, 9 kHz frequency is chosen from that spectrum to model the signals, as well as to keep a low computation time. Simulations have been performed using MATLAB, where two signals were created – a direct signal $S_1(t)$ and a multipath signal $S_2(t)$. The multipath signal has an additional dampening factor on the amplitude and a phase delay. These were then added to create the received signal $S_r(t)$, as shown in the equations below:

$$\text{Direct signal: } S_1(t) = \alpha\cos(\varphi) \quad \text{where } \alpha = 1, \varphi = 2\pi f t \tag{1}$$

$$\text{Multipath signal: } S_2(t) = \beta\cos(\varphi + \Delta\varphi) \quad \text{where } \beta = 0.4861,\ \Delta\varphi = \frac{\pi}{2} \tag{2}$$

$$\text{Resultant signal: } S_r(t) = S_1(t) + S_2(t) \tag{3}$$
$$S_r(t) = \alpha\cos(\varphi) + \beta\cos(\varphi + \Delta\varphi)$$

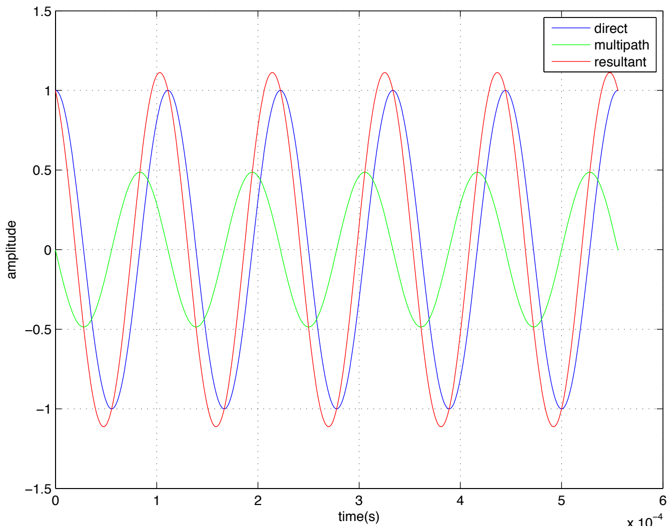

Fig. 2. A plot of the direct signal, multipath signal, and the resultant signal formed from combining the direct and multipath signal. 5 periods of the signals are plotted.

III. Technical Approach: Filter Design

The received signal $S_r(t)$ is used as the input to the LMS filter. The LMS filter configuration used in this case is the transversal LMS filter. The weight coefficients are adaptively adjusted by using the error signal as a feedback to the system.

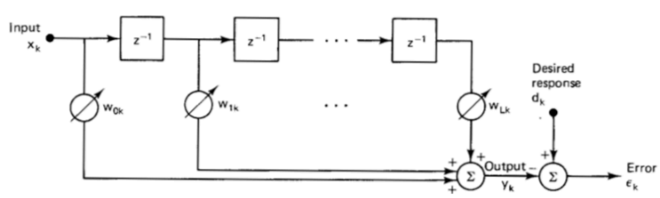

Fig. 3. A transversal LMS filter model.

The step size is defined using the signal power and the number of weights used in the filter:

$$\text{Step Size: } 0 < \mu < \frac{1}{(L+1)(Signal\ Power)} \tag{4}$$

The signal power of the received signal is estimated by using the root mean square of its amplitude value, 0.5 in this case. L+1 represents the number of filter taps (weights).

The received signal is delayed and stored into a large vector X. The output is calculated by multiplying the vector X with the weight vector:

$$y_k = X_k^T W_k \tag{5}$$

The error signal is then calculated by subtracting the output from the desired signal:

$$\epsilon_k = d_k - y_k \tag{6}$$

The weight vectors are updated for the next iteration by multiplying the weights of the current iteration with the step size, error signal and the vector X:

$$W_{k+1} = W_k + 2\mu\varepsilon_k X_k \tag{7}$$

The error signal is squared to use as an estimate for the Mean Squared Error (MSE). These calculations are carried out in a loop for the entire number of samples of the input signal, for each of the weights of the filter. The ratio of the sampling frequency to the input signal frequency, and the number of periods of the input signal to be used in the simulation, determines the number of iteration for each weight.

IV. Simulations

The first set of simulations that are going to be looked at is carried out with the pair of input signals set at 9 kHz, and the sampling frequency set at 900 kHz for a period of 5 complete waveforms. This will set the number of iterations to 502. First, a 2 tap LMS filter is used and the outputs are analyzed.

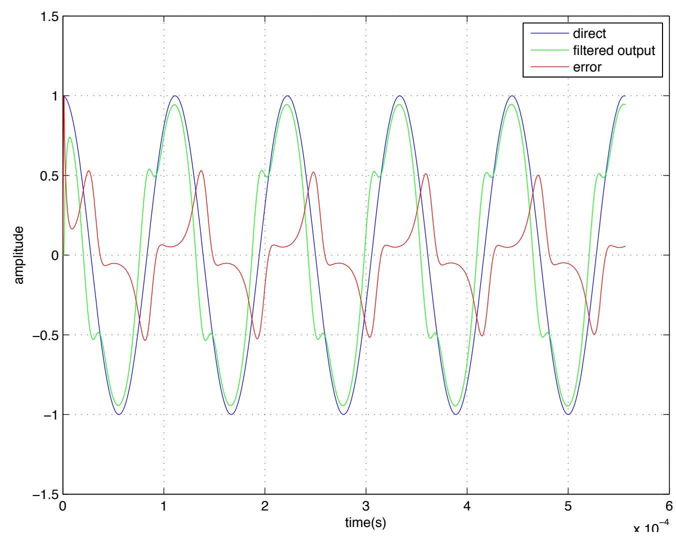

Fig. 4. Two tap LMS filtered output of the input signal

It can be observed that the error signal is consistent and does not die out in the 5 periods plotted. The output signal still has a significant amount of damping and phase delay.

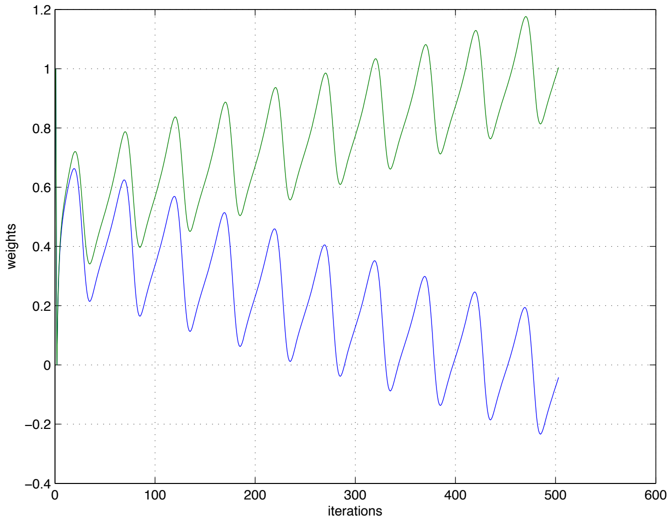

Fig. 5. Adaptation of the weights of the two tap LMS filter against number of iterations

The weights are oscillating constantly, as well as diverging. This behavior is indicative of the fact that the filter will not be able to reach optimum values for the weights where the error will be minimized.

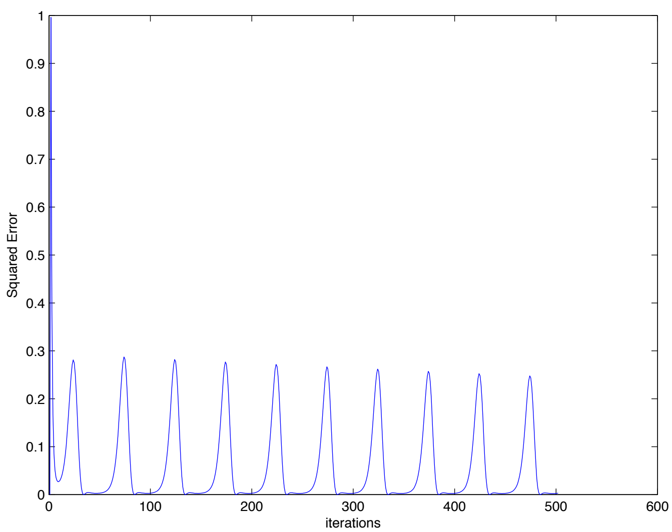

Fig. 6. Squared Error of the two tap LMS filter, used as an estimate for Mean Squared Error, against the number of iterations

The squared error curve is also oscillating at a consistent rate with a very consistent peak level. This indicates that the mean squared error will probably stay the same even after a lot of iterations.

Next, a different variant of the LMS filter is studied, the NLMS filter. Its performance is evaluated with the same input. The main difference between the implementation of the two filters is how the weights are updated at each iteration:

$$W_{k+1} = W_k + 2\mu\varepsilon_k \frac{X_k}{X_k^T X_k} \qquad (8)$$

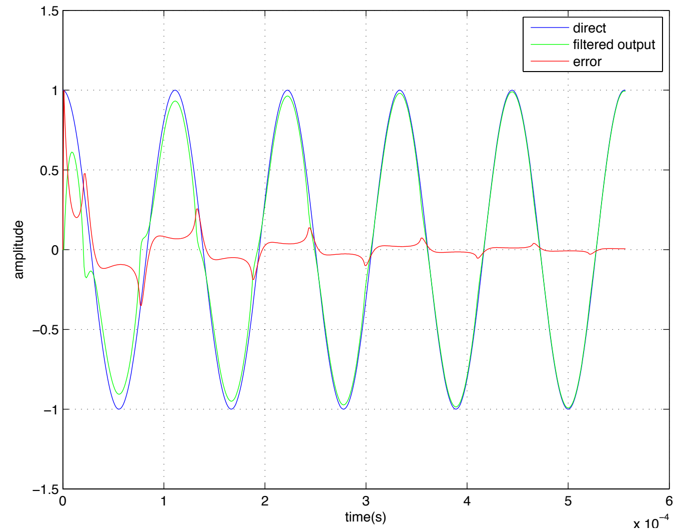

Fig. 7. Two tap NLMS filtered output of the input signal

Compared to the LMS filter, the NLMS filter performs exceedingly well. The error decreases quickly to almost a negligible level. The output signal almost overlaps with the direct signal.

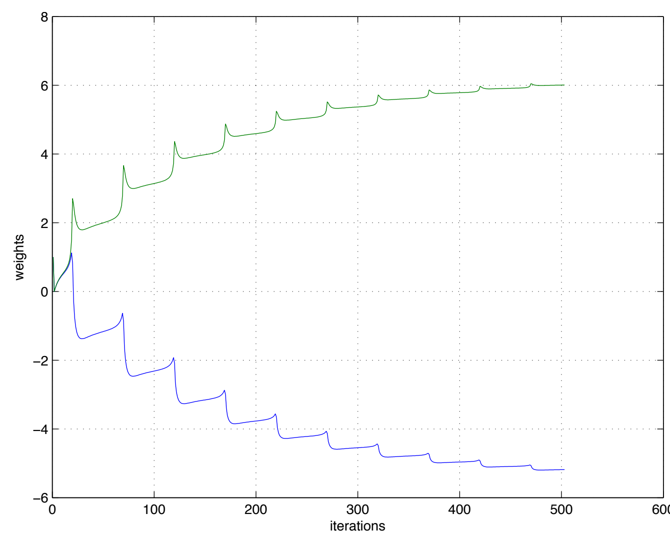

Fig. 8. Adaptation of the weights of the two tap NLMS filter against the number of iterations

The weights initially oscillate but fall off eventually and decreases in the level of variation from peak to peak. They eventually slow down, showing signs of converging to a steady state value.

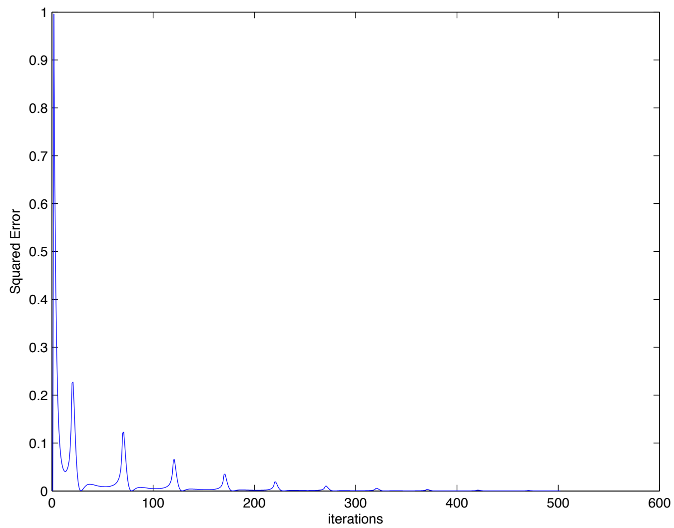

Fig. 9. Squared Error of the two tap NLMS filter, used as an estimate for Mean Squared Error, against the number of iteration

The squared error curve falls to negligible levels around 300 iterations, showing very good performance, in a very short time.

The next set of simulations that are going to be looked at are done with an 8 tap LMS filter. The rest of the parameters have been kept the same to ensure a fair comparison.

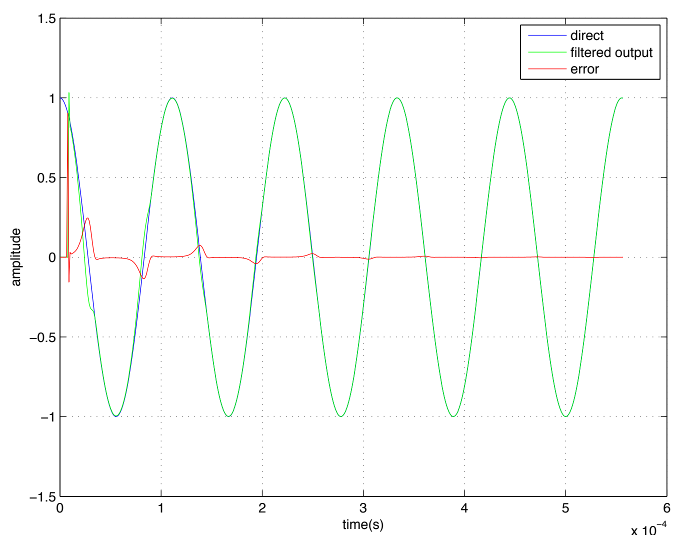

Fig. 10. Eight tap LMS filtered output of the input signal

With eight taps, it can be seen that the LMS algorithm is sufficient enough to suppress the error signal to negligible levels in a very short period of time (not visible after 3 seconds). The output signal and the input signal virtually overlap, proving the effectiveness of using the LMS algorithm in this instance.

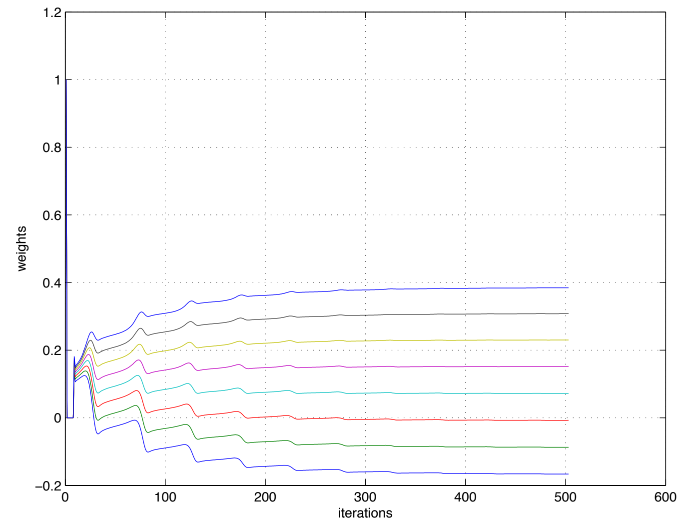

Fig. 11. Adaptation of the weights of the eight tap LMS filter against number of iterations

In contrast to the two tap LMS filter where the weights diverged, the weights of the eight tap LMS filter converge to a steady state value, around 300 iterations. This is proves that the filter reaches optimum weight values with increased number of weights, keeping the error minimized to its lowest possible value by the filter.

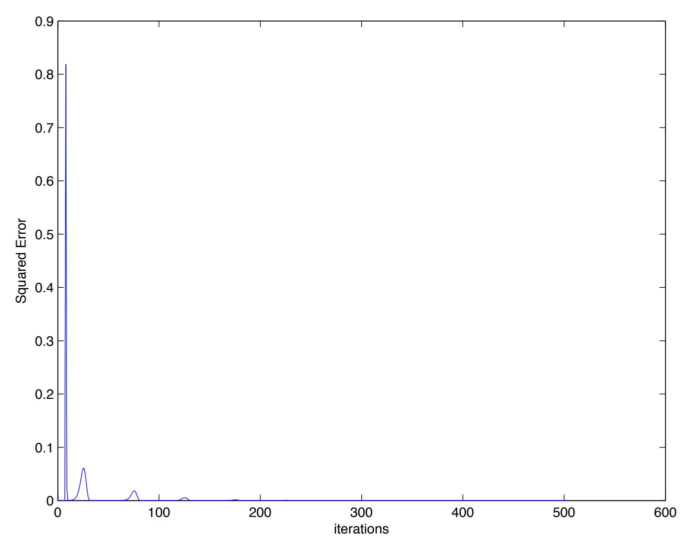

Fig. 12. Squared Error of the eight tap LMS filter, used as an estimate for Mean Squared Error, against iterations

The squared error curve also proves to be satisfactory in this instance – the curve falls to zero very quickly, with virtually no peaks after 200 iterations.

The performance of the LMS filter is then evaluated by increasing the number of iterations for each weight. This is achieved by increasing the sampling rate from 900 kHz to 9000 kHz, which in turn shifts the number of iterations from 502 to 5002.

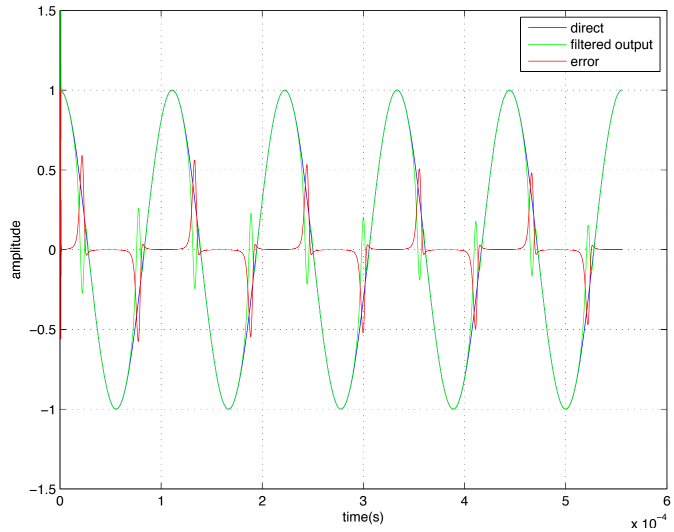

Fig. 13. Eight tap LMS filtered output of the input signal, against increased (5002) iterations

From Fig. 13, it can be observed that an increased sampling rate has a detrimental effect on the filtered output. The error spikes up before the start and end of every crest. Other than that however, for the rest of the signal, the error is mostly filtered out and the direct signal and filtered output greatly overlap.

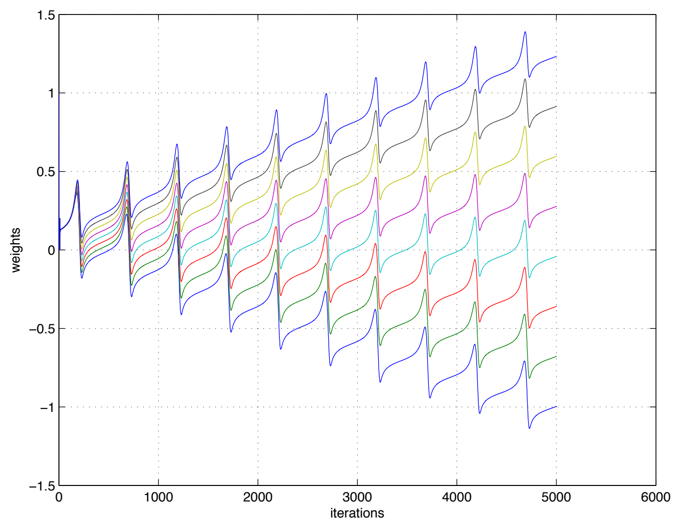

Fig. 14. Adaptation of the weights of the eight tap LMS filter against increased number of iterations (5002)

Weights oscillate at higher levels of iterations, and also do not indicate any sign of convergence. Unlike the simulation with 502 iterations for LMS filter, the filter in this case does not seem likely to reach optimum weight values and a steady state convergence with 5002 iterations.

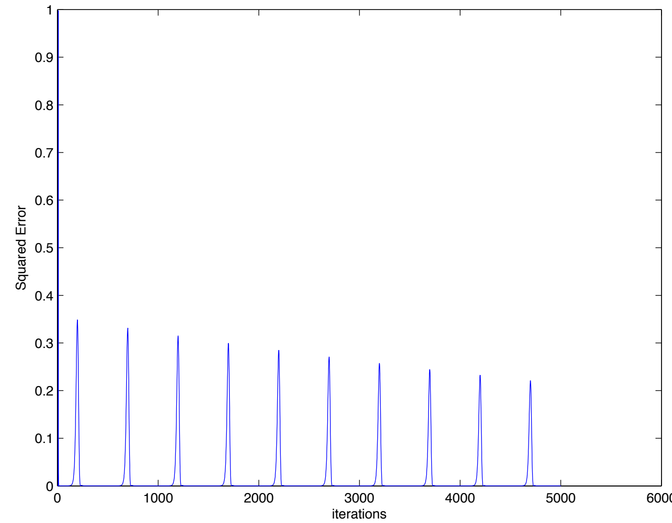

Fig. 15. Squared Error of the eight tap LMS filter, used as an estimate for Mean Squared Error, against increased (5002) iterations

The squared error curve resembles that of the two tap filter's squared error curve – an oscillating pattern, except with a decreasing peak value. This indicates that the curve will require a lot more iterations to go down to an acceptable level.

Keeping the parameters the same along with the increased iteration number (5002) and 8 taps, the performance for NLMS is now analyzed.

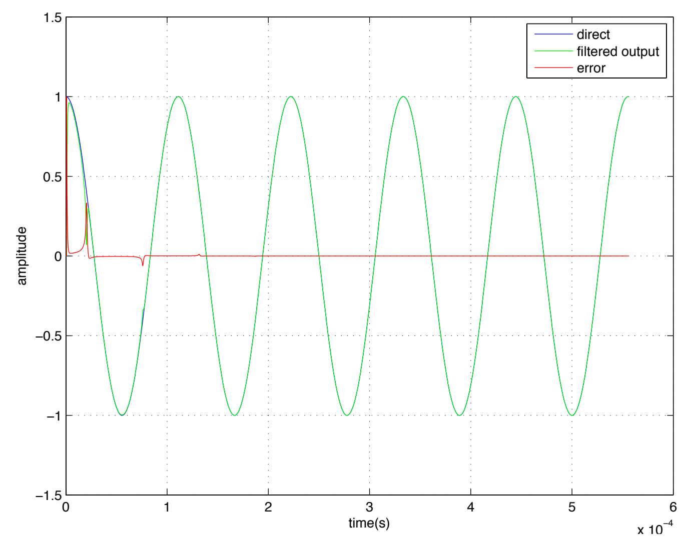

Fig. 16. Eight tap NLMS filtered output of the input signal, against increased (5002) iterations

Staying consistent with previous observations, NLMS provided much better results, with the error level falling to negligible levels very quickly – virtually non-existent around 2 seconds.

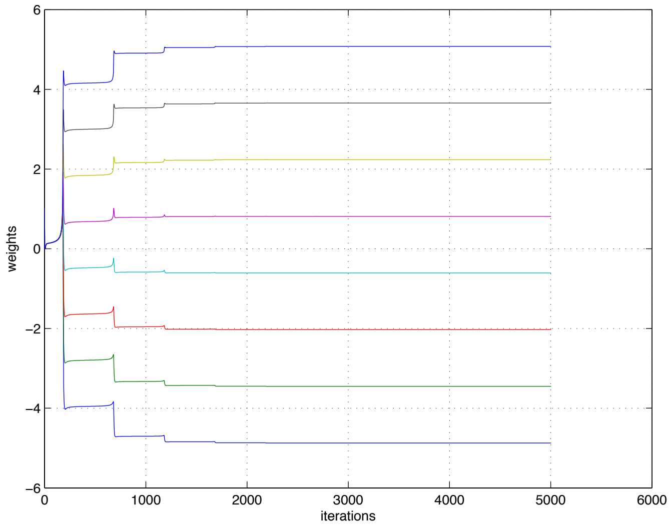

Fig. 17. Adaptation of the weights of the eight tap NLMS filter against increased number of iterations (5002)

The weights of the NLMS filter converge really quickly, and within 2000 iterations they reach a steady state value. This is much better performance in comparison to the LMS filter. It can be deduced from this observation that the errors are minimized to its lowest possible value without any variations by 2000 iterations.

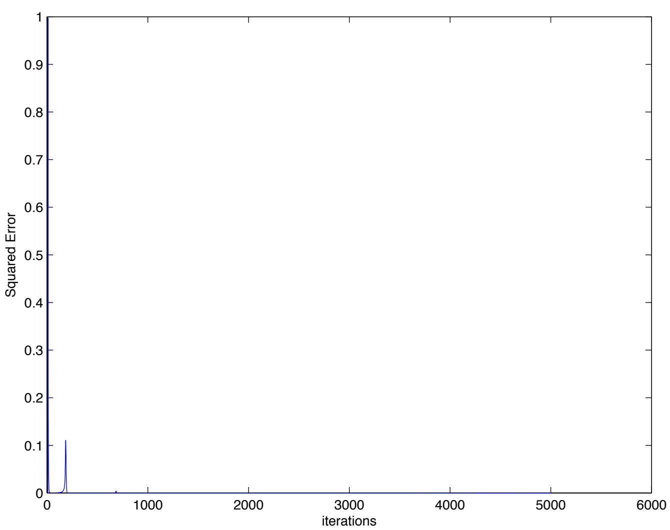

Fig. 18. Squared Error of the eight tap NLMS filter, used as an estimate for Mean Squared Error, against increased (5002) iterations

From Fig. 18, the squared error shows very satisfactory performance, with a single peak and then dropping to zero.

V. Applications

Applications of the LMS and NLMS algorithms presented in this paper include incorporating them in systems where the data is static, like GPS systems. The fast convergence rate and the low computational complexity make it a very convenient and attractive approach for mitigating multipath. According to [2], real world GPS signals were used to test a 32 tap LMS filter modelled using the Code Minus Carrier (CMC) method, which showed a 99.8% reduction in multipath error, backing up the observations from the simulation data presented in this paper.

VI. Summary

Based on the results of the output of the simulations, multipath mitigation using LMS filtering showed promising results. For the two tap filter, the NLMS variant proved to have a much better performance compared to the LMS filter. However, the error was still visible in the output. Performance improved when the number of taps was increased from two to eight. With the eight tap filter, both LMS and NLMS filter showed great performance; the error signal completely fell down to zero. Finally, with a higher sampling frequency, the weights of the eight tap LMS filter exhibited an oscillating divergent pattern and a consistent error at the start and end of every crest of the signal. This problem was not faced by the eight tap NLMS filter, where the weights were convergent and the error went to zero at a relatively small number of iterations. Overall, the NLMS filter proved to be better than the LMS filter to mitigate multipath.

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